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HEAT EXCHANGE DURING FREE MOVEMENT AROUND  
A HORIZONTAL CYLINDER IN RAREFIED AIR

By

A.K. Rebrov

## **UNEDITED ROUGH DRAFT TRANSLATION**

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**BY: A.K. Rebrov**

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# Heat Exchange during Free Movement around a Horizontal Cylinder in Rarefied Air

by

A.K.Rebrov

In this report are given the results obtained during experimental investigating heat exchange during free movement of air in rarefied space. Criterial relations are given for the calculation of heat exchange in a broad range of Gr Pr values, and in the presence of a substantial temperature jump as well.

At present time reliable generalized dependences for heat exchange at free movement have been obtained for turbulent conditions and for such conditions of laminary movement, when the thickness of the boundary layer is small in comparison to the dimensions of the body.

The theory of similarity allows to establish, that the determinant criterion for heat exchange under conditions of laminary free movement, in the case of negligibly low effect of inertia forces are the derivatives of the Grashof and Prandtl Gr Pr criteria. According to numerous experimental data, at small Gr Pr values (of the order of 1 and less) the formation of the hydrodynamic and thermal boundary layer is greatly influenced by the dimensions and form of the body. This characteristic can take place at small dimensions of the body or at low pressures. The lack of generally accepted mathematical dependencies calls for special investigation of heat exchange with consideration of the factors mentioned. In this experiment is investigated heat exchange of a horizontal cylinder in rarefied air. Special attention is being devoted to the effect on heat exchange of the temperature jump at the

at the surface of the sample.

Heat exchange of a horizontal cylinder was investigated by M/A. Mikheyev [1], Klenbas [2], Senftleben [3]. In the experiments by Madden and Piret [4], Kyte, Madden and Piret [5] are presented data of experiments in rarefied gases.

The broadest scope is taken up by criterial dependencies  $Nu = f(Gr Pr)$  obtained in experiments [3] and [5]. In fig 1 are given curves corresponding to the Senftleben formula

$$Nu = \frac{2}{\ln s} \left[ 1 - \frac{0,033}{(Gr Pr)^{\frac{1}{4}} \ln s} \left[ \sqrt{1 + \frac{(Gr Pr)^{\frac{1}{4}} \ln s}{0,033}} - 1 \right] \right] \quad (1)$$

at  $s = 1 + \frac{4,5}{(Gr Pr)^{\frac{1}{4}}}$  for  $10^{-5} < Gr Pr < 10^8$

and Kyte, Madden Piret equations

$$\exp \frac{2}{Nu} - 1 = \frac{7,09}{(Gr Pr)^{0,37}} \text{ for } 10^{-7} < Gr Pr < 10^{1,5}, \quad (2)$$

$$\exp \frac{2}{Nu} - 1 = \frac{5,01}{(Gr Pr)^{0,25}} \text{ for } 10^{1,5} < Gr Pr < 10^9. \quad (3)$$

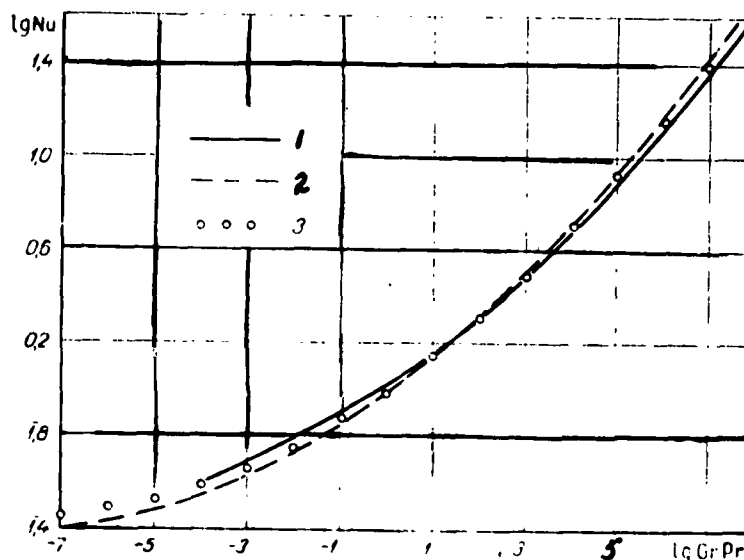


Fig. 1. Heat exchange of a horizontal cylinder in unlimited space  
1-Senftleben dependence; 2-Kyte, Madden Piret dependence; 3-generalizations of present report.

Equations (2) and (3) are valid in the absence on the surface of a temperature jump and slide of effects - caused by deep rarefaction.

A comparison of dependences on fig.1, does not reveal the effect of rarefaction on the laws of heat exchange during free movement in a wide range of Gr Pr numbers if the near-wall rarefaction effects are not substantial.

The authors [4] and [5] conducted experiments with thin wires with diameter of 0.07, 0.078 and 0.251 mm. The tested species were placed in a hollow under a glass cover with diameter of 457 and height of 660 mm. The pressure was changed within limits of 0.05 mm Hg to one atm. The dependencies obtained by these authors can be accepted in the case when there is no shell effect on the boundary layer. This problem was baseless in the reports by [4] and [5]. Since at low pressure the boundary layer dimensions can be very great, and the process of heat exchange in a rarefied gas under real conditions, is on a limited scale, the effect of the walls is a possibility. Under definite conditions, as will be shown below, it may become quite substantial.

To study the above mentioned characteristics and establish the boundaries and nature of the influences of the rarefaction effects, particularly the effect of the temperature jump, heat exchange was investigated of horizontal cylinders with diameters of 1.31 mm (made of stainless steel) and 9.9 mm (made of copper) in a pressure range from 0.005 to 130 mm Hg and temperatures of from 50 to 150°C. The specimens were placed in the center of a steel cylinder-shell with diameter of 520 mm height of 600 mm parallel to bottom and lid.

The temperature of the cylinders was measured with the aid of Nichrome-Constantan thermocouples and semiautomatic potentiometer type P2/1. To measure pressures in the range of 0.001-5 mm Hg was used the McLeod multirange pressure gage, in

the 5-130 mm Hg range with a shortened U-shaped mercury pressure gage.

When processing the experimental results in oriterial form the determinant dimension was the diameter of the cylinder; the physical parameters of the air were established by the average temperature of the cylinder and shell.

The intensity of heating the sample was found with the aid of ammeters of the 0.2 class and P2/1 potentiometer. Heat losses due to emission and heat conduction according to the thermocouple wires were estimated at thorough evacuation of the system - to pressures of the order of  $2 \cdot 10^{-5}$  mm Hg.

On fig.2 are given the experimental dependencies  $Nu=f(Gr Pr)$  for the investigated range of pressures. The nature of the dependencies has certain qualities, described in report [6] where a partial investigation was made of the problem concerning the effect of rarefaction on heat transfer in space between coaxial cylinders.

In the drawing are shown sections, where the value  $Nu$  does practically not depend upon the derivative  $GrPr$  (for samples with diameter of 9.9 mm at  $Gr Pr = 10^{-1.5} - 10^{-4}$ , with diameter of 1.31 mm at  $Gr Pr = 10^{-5} - 10^{-6}$ ). This indicates a reduction in the effect of free movement on the heat exchange because of contraction of the boundary layer, caused by the reverse currents near the walls of the shell.

In fig.2 are also given data of experiments made by [4]. The absence among them of the heat exchange condition, close to pure heat conduction, is explained by the fact, that for thin wires the dimensions of the boundary layer were small in comparison to the dimensions of a bell and there was no wall effect.

Following the condition of almost pure heat conduction the sharp drop in heat exchange is caused by the rise in temperature jump at the wall of the specimen. The temperature jump becomes substantial at such pressures, when the average length of molecular free run becomes compatible with the dimensions of the body. This phenomenon was investigated by [7][8] and others.



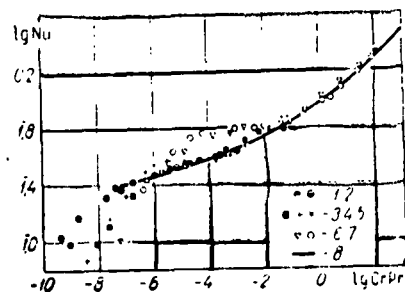


Fig. 2. Experimental data on heat transfer and presented here are well superimposed of horizontal cylinders in rarefied air

1,2-Madden First at  $d = 0.07, 0.251$  mm,  $t = 65^\circ$ ; 3,4,5 - the authors' at  $t = 150$ , and where the temperature jump has not yet 100,  $50^\circ$ C,  $d = 1.31$  mm; 6,7 - the authors' at  $t = 150, 100^\circ$ C,  $d = 9.9$  mm; 8 - by dependence taken effect. The dependence for that curve (7).

At greater rarefactions the effect of the temperature jump leads to a sharp scattering of dependence points  $Nu = f(GrPr)$ . Consequently, to represent the experimental data by a general criterial equation one determinant  $GrPr$  criterion is insufficient.

By examining fig 2 it becomes clear, that the data of the American investigators [4]

on one curve, where there is no wall effect

is nonlinear in logarithmic coordinates. It

can be expressed in form of

$$Nu_K = C (GrPr)^n \quad (4)$$

Here and henceforth  $Nu_K$  designates the Nusselt criterion for heat exchange at free movement in unlimited space;  $C$  and  $n$  - variables, synonymously determinable by the  $GrPr$  derivative.

The dependence for  $n$  according to experimental results in the range  $10^{-7} < GrPr < 10^3$  is obtained rectilinear and is expressed by formula

$$n = 0.14 + 0.015 \lg GrPr \quad (5)$$

In this very range

$$C = 0.98 - 0.01 (\lg GrPr)^2 \quad (6)$$

In this way equation (4) for heat exchange at laminary free movement around a horizontal cylinder in unlimited space acquires the form of

$$Nu_K = [0.98 - 0.01 (\lg GrPr)^2] (GrPr)^{0.14+0.015 \lg GrPr} \quad (7)$$

The derived formula is in excellent conformity with the dependencies, presented in fig.1. It can be perfectly well used in the range of  $10^{-7} < GrPr < 10^8$ .

The lower intensity limit of heat exchange at free movement is the transfer of heat by heat conduction. Analysis of results of experiments by [3] [4] [5] and experiments carried out here allow to state, that for the horizontal cylinder in unlimited space or in the absence of shell wall effect such a limit does not come into being even at very small GrPr numbers.

In limited space at very small GrPr values heat exchange will be determined by pure heat conduction. In the simpler case, e.g. for two coaxial cylinders of infinite length, the Nu criterion for heat conduction will be

$$Nu_T = \frac{2}{\ln d_2/d_1} \quad (8)$$

As is evident, the maximum value of the Nu number depends upon the ratio of the diameters and has no constant value. This is true also for vertical cylinders. Therefore the remarks of certain authors [9] about the existence of a limit in  $Nu_T$  number at heat exchange under conditions of free movement for a cylinder can be considered as void of any foundations.

When calculating heat exchange by formula (7) are possible errors, the sources of which are:

- a) thermal losses along the axis of the specimen as result of its finite length;
- b) effect of shell walls on boundary layer;
- c) effect of temperature jump at surface of sample;

We shall establish the limits of applicability of this formula.

If the cylindrical sample is in limited space, the the heat exchange conditions for it are somewhat close to the heat exchange conditions for a sphere. For two concentric spheres with diameters  $d_1$  and  $d_2$  the  $Nu_T$  number, determinable by the dia -

meter of the inner sphere  $d_1$ , will be found from formula

$$Nu_T = \frac{2}{1 - d_1/d_2} \quad (9)$$

within the limit, when  $d_1/d_2 \rightarrow 0, Nu_T \rightarrow 2$ .

If the radius of the sphere  $r_2$  is equal to the distance from the axis of the cylindrical sample to the shell, and the diameter of sphere  $d_1$  equals the diameter of the sample, then the value  $Nu_T$  for the cylinder of finite dimensions lies between the results, obtained by formulas (8) and (9). This value appears to be the maximum limit of existence of a free movement around a cylinder in limited space.

$Nu_T$  can be determined exactly, by solving the problem of heat conduction in bodies of complex form.  $Nu_T$  is found approximately by formula (8) for species with  $\frac{d}{L} \leq 0.01$  ( $L$  - length of specimen)

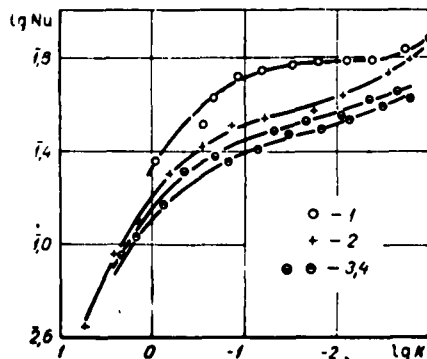
If  $Nu_T$  is known, then the maximum value  $GrPr_T$ , at which formula (7) is still applicable, can be found either by fig.2 or directly from formula (7).

The change over into a condition of almost pure heat conduction is quite abrupt, it does not depend, by the way, upon the temperature of the specimen. This adds a certain definiteness into using the maximum  $GrPr$  value. At  $GrPr < (GrPr)_T$  values the calculation must be conducted by heat conduction formulas. The use of (7) for these conditions may lead to greater errors.

According to carried out experiments (fig.2) the effect of shell walls can be considered if  $GrPr \geq 10^{1.5}$  at  $d_1/d_2 \leq 1.8 \cdot 10^{-2}$ , and also when  $GrPr \geq 10^{-5}$  at  $d_1/d_2 \leq 2 \cdot 10^{-3}$ . As  $d_2$  was accepted a doubled average distance from the axis of the specimen to the shell.

The effect of the temperature jump, as established by experiments, may reflect itself in the condition of pure heat conduction as well as in the condition of undistorted by walls free motion. The determinant criterion for consideration of the temperature jump is the Knudsen number  $K = \bar{l}/d$ , where  $\bar{l}$  - average length of free

run of molecules, and  $d$ -diameter of the cylinder.



**Fig. 3. Dependence  $\lg Nu$  upon  $\lg K$ .**  
and 2-according to experiments by the author at  $d = 9.9$  mm and  $1.31$  mm,  $t = 100^\circ\text{C}$ ; 3 & 4 according to Madden Piret experiments at  $d = 0.251$  mm and  $0.07$  mm,  $t = 65^\circ\text{C}$ .

On fig. 3 in logarithmic coordinates are given the dependences  $Nu = f(K)$  in accordance with experimental results obtained by the author and other researchers [4]. The beginning of the near-wall rarefaction effect is determined by curve inflection points, which for a cylinder with a 9.9 mm diameter corresponds to a condition of almost pure heat conduction ( $\lg Gr Pr \approx -3$ , fig. 2), for cylinders of small dimensions (Madden Piret experiments) - to a condition of free movement ( $\lg Gr Pr = -7$ ). In all instances the indicated effect begins at  $\lg K \approx 2.3$ , i.e. at  $K \approx 0.02$ .

For conditions of heat transfer by heat conduction at pressures, when the temperature jump at the wall reduced heat exchange ( $K > 0.02$ ), instead of formula (8) for coaxial cylinders it is easy to obtain

$$Nu_p = \frac{2}{\ln \frac{d_2}{d_1} + 2 \zeta \frac{l}{d_1} \left( \frac{d_1}{d_2} + 1 \right)} \quad (11)$$

$Nu_p$  will designate the Nusselt criterion for heat exchange in the presence of a substantial temperature jump.

In deriving formula (11) was used an expression for the temperature jump at the

Footnote to page 7....\* The magnitude  $I$  for air was determined by formula obtained from the dependence in the book by Dashman [7]:  $I = 1563 \frac{\mu}{p} \sqrt{T_{av}} \quad (10)$  where  $\mu$  - dynamic strength,  $\text{kg. sec/m}^2$ ;  $p$  - pressure in mm Hg;  $T_{av}$  - average temperature of air,  $^\circ\text{K}$ .

surface of a cylinder in form of

$$\Delta T = \beta \int \Delta T / dr \quad (12)$$

Here, according to Kennard [7]

$$\beta = \frac{2-a}{a} \frac{2}{k+1} \frac{9k-5}{4}, \quad (13)$$

where  $a$  - coefficient of thermal accommodation - value, characterizing the completeness of molecular energy exchange on the surface;  $k = c_p/c_v$ . The accommodation coefficients on the surface of the cylinder and shell for the sake of simplicity were accepted as identical.

Having noticed that  $\bar{l}/d_1 = K$ , and the magnitude  $d_1/d_2 + 1$  at small  $d_1/d_2$  can be considered approximately equal to 1, we can write

$$Nu_p = \frac{2}{\ln \frac{d_2}{d_1} + 2\beta K} \quad (14)$$

Taking into consideration (8) we obtain

$$Nu_p = \frac{1}{\frac{1}{Nu_r} + \beta K} \quad (15)$$

To calculate the transfer of heat by heat conduction from a cylinder of finite dimensions in rarefied air the value  $Nu_T$  is determined for limited space, as was shown before.

Kavanau [10] obtained a formula in the form of (15), describing heat exchange of a sphere in a subsonic flow of rarefied air. In making a conclusion he assumed that the heat, transmitted from a surface at a temperature  $t_w$  to a gas at a temperature  $t_1$ , for the case of a temperature jump equalling the heat, which would be transmitted at a temperature difference of the surface and gas  $(t_w - \Delta t) - t_1$  and absence of temperature jump.

According to Kavanau, beta can be found from experiment. Determined by experimen-

tation, beta takes under consideration the errors from simplifying assumptions during the derivation of formula (14). These errors can be insignificant.

From formula (15)

$$\frac{1}{Nu_p} - \frac{1}{Nu_T} = \beta K. \quad (16)$$

Dependence (16) shown in fig 4 in accordance with experimental results with copper cylinder ( $d_1=9.9$  mm) and with cylinder made of stainless steel ( $d_1=1.31$  mm). The value  $Nu_T$  was accepted for both instances according to the curve on fig.2.

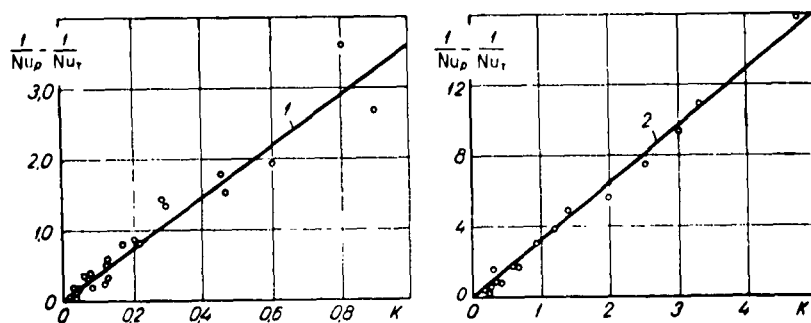


Fig.4. Dependences  $\frac{1}{Nu_p} - \frac{1}{Nu_T} = f(K)$ :

1- for copper cylinder; 2- for stainless steel cylinder

The obtained dependences (functions) are rectilinear; to them correspond values  $\beta = 3.5$  for the copper cylinder and 3.28 for the stainless steel cylinder. The experimental data were processed by the method of the least squares. If the errors in determining beta are disregarded, then for the coefficient of accommodation  $\alpha$  are obtained values of 0.65 and 0.67 respectively.

At very high rarefactions, when the length of the free run will be equal or greater than the dimensions of the shell, the concept about the temperature jump and about the transmission of heat by heat conduction, loses its meaning. Heat transfer

is realized by free molecules, having no intermediate collisions. In formula (15) in this case can be disregarded the first member in the denominator, thus arriving at a dependence for free molecular heat exchange. This phenomenon has been well investigated. The mathematical formulas are applied in courses of molecular-kinetic theory of gases.

To calculate heat exchange at a developed laminary free movement with consideration of the temperature jump it is advisable to use formula

$$Nu_p = \frac{1}{Nu_k + \beta K} \quad (17)$$

where  $Nu_k$  - Nusselt criterion, found for free movement from expression (7). The effect of the temperature jump is considered as the second member in the denominator.

Formula (17) is valid for a body of any given form with constant temperature over the surface. It is obtained easily when investigating heat exchange on the surface in conditions of rarefaction, when it is necessary to take into consideration the temperature jump. When  $K < 0.02$  formula (17) is brought down to (7), provided the value  $\beta K$  is disregarded.

#### Summary in English language

#### Literature

1. Mikheyev, M.A. Bases of Heat Transfer, GOSENERGOIZDAT, 1956
  - 2, 4, 5 and 10 of English origin.
  3. Senftleben, H. Journ. of Appl. Physics 3, 361-372, 1951
  6. Mikheyev, M.A. Zhurnal Teoreticheskoy Fiziki, 3, 5, 698-711, 1933.
  7. Deshman, S. Scientific Principles of Vacuum Technology, Foreign Liter. 1950
  8. Lazarev, P.O; Compositions, vol.2. Published by Acad. of Sc. USSR, 1950
  9. Buznik, V.M; Vozlomnyy, A.K; Izvestiya Vuzov Energetika, No 2, 1960
- Jan. 1. 1961. Aviation Inst. city of Kazan'

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